Calculation of Age-conditional Probability of Dying of A Specific Cancer

To calculate the age-conditional probability of dying of a specific cancer we use standard competing risks methodology (see e.g., Kalbfleisch and Prentice, 1980). First, we repeat (almost identically) the description of the problem from the beginning of Section 2 of Fay, Pfeiffer, Cronin, Le, and Feuer (2003):

We observe the time until one of several events, \( T \), and an indicator of the type of event that occurred, \( J \). [The value] \( T \) is a random variable denoting the age at death and \( J \) has one of 2 values, \( J = d \) means death from the event of interest (e.g., breast cancer), and \( J = o \) means death from other causes. For ease of exposition, we use the term “cancer” to denote the event of interest. The cause specific hazard function for \( J = j \) is

\[
\lambda_j(a) = \lim_{\epsilon \to 0^+} \frac{Pr[a \leq T < a + \epsilon, J = j|T \geq a]}{\epsilon}.
\]

Thus \( \lambda_d(a) \) is the rate of cancer deaths per person-years alive at age \( a \), and \( \lambda_o(a) \) is the rate of other (i.e., non-cancer) deaths per person-years alive at age \( a \). The overall failure rate at age \( a \) is \( \lambda(a) = \lambda_d(a) + \lambda_o(a) \), and the overall survival function is \( S(a) = Pr[T > a] = \exp\left( -\int_0^a \lambda(u)du \right) \). The probability of dying from cause \( j \) in the age interval \([x, y)\) given survival until just prior to \( x \) is

\[
Pr[x \leq T < y, J = j|T \geq x] = \frac{\int_x^y \lambda_j(u)S(u-)du}{S(x-)}.
\]

where \( S(a-) = \lim_{\epsilon \to 0} S(a - \epsilon) \).

Thus, the result depends on the method for estimating the \( \lambda_d(a) \) and \( \lambda_o(a) \) functions. In version 5.0 of DevCan, we use the piecewise mid-age joinpoint model for the rates described in Fay (2003), where the pieces are \( \frac{1}{2} \) year long. Previous versions of the DevCan software used the simple 5-year piecewise constant rate model described in Fay, et al. (2003). The method for confidence interval estimation is described in Fay, et al. (2003).

Note: The previous description from the DevCan website for the calculation of the age-conditional probability of dying used notation similar to that used in multiple decrement life tables (see e.g., Elandt-Johnson and Johnson, 1980). Although the notations look very different, the calculations are the same. This is not true for the calculation of the age-conditional probability of developing cancer, where
the new method described in Fay, et al. (2003) is different from the method described in Wun, Merrrill, and Feuer (1998), even though both methods use the simple 5-year piecewise constant rate model in their calculations (see Fay, et al., 2002).

References


